[This question paper contains 6 printed pages.]

(15) **Your Roll No.** 2022

Sr. No. of Question Paper : 1152 **A**

Unique Paper Code : 32221401

Name of the Paper : Mathematical Physics – III

Name of the Course : **B.Sc. (Hons.) Physics**

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **five** questions in all.
3. Question No. **1** is compulsory.

1. Attempt any **five** parts : (5×3=15)

   (a) Find the cube root of $z = 1 + i$ and locate them in the plane.

   (b) Show that $u(x, y) = x^2 - y^2$ is a harmonic function in the whole complex plane, find its harmonic conjugate, $v(x, y)$.

P.T.O.

(c) Evaluate

$$\oint_C \frac{e^{3z}}{(z - i\,\pi)}\,dz \qquad C : |z-1| = 4.$$

(d) If a complex function f(z) is analytic in a domain D and |f(z)| = Const. K in D, then show that f(z) is also constant in D.

(e) Show that the Laplace Transform of Dirac delta function is 1, ie, $L\{\delta(t)\} = 1$.

(f) If Laplace Transform of a function $L\{f(t)\} = F(s)$, show that

$$L\{t^n f(t)\} = (-1)^n\, F^{(n)}(s),$$

where $F^{(n)}$ represents n-th derivative of F(s).

(g) If Fourier Transform of f(x) is $F(\omega)$, find Fourier Transform of f(x) cos ax, where $a > 0$.

(h) Evaluate the following integrals

(i) $\int_0^\infty e^{3t}\,\delta(t-4)\,dt$

(ii) $\int_0^\infty \sin 2t\,\delta(t-\pi/4)\,dt.$

2. (a) Find all values of $\sin^{-1} 2$. (5)

(b) Expand $f(z) = e^{z/(z-2)}$ in the Laurent series about $z = 2$ and determine the region of convergence of this series. Also classify the singularity. (6)

(c) Evaluate (4)

$$\oint_C \frac{z}{z^2+9}\,dz, \quad \text{where} \quad C : |z-2i| = 4$$

3. Using Contour Integration, solve any **two** of the followings : (7.5×2=15)

(a) $\int_0^\infty \frac{dx}{x^4+1}$

(b) $\int_0^\pi \frac{a}{a^2+\sin^2\theta}\,d\theta \quad a > 0$

(c) $\int_0^\infty \frac{\cos x\,dx}{(x^2+a^2)(x^2+b^2)} \quad a \,\&b\, > 0$

(d) $\int_0^\infty \frac{\sin^2 x}{x^2}\,dx$

4. (a) Obtain Fourier Integral representation of the function (4)

$$f(x) = \begin{cases} 0 & x < 0 \\ a & 0 \leq x \leq 3 \\ 0 & x > 3 \end{cases}$$

(b) Find the Fourier Transform of the function (4)

$$f(x) = \frac{x}{x^2 + 1}$$

(c) Find Fourier sine transform of e^{-mx}, $m > 0$ and hence evaluate the integral (7)

$$\int_0^\infty \frac{\omega \sin \omega x}{a^2 + \omega^2} \, d\omega.$$

5. (a) For a periodic function f(t) having periodicity T, such that $f(t + T) = f(t)$, show that the Laplace Transform is given by (7)

$$L\{f(t)\} = \frac{\int_0^T e^{-st} f(t)\, dt}{1 - e^{sT}}.$$

(b) If a function is piece-wise continuous on $0 < t \leq T$ and is of exponential order for $t > T$ then show that (8)

$$\lim_{s\to\infty} L\{f(t)\} = \lim_{s\to\infty} F(s) = 0,$$

and hence further show that

$$\lim_{t\to 0} f(t) = \lim_{s\to\infty} sF(s)$$

where F(s) represents the Laplace Transform of f(t).

6. (a) Plot the given function (5)

$$f(x) = \begin{cases} 1 & |x| < 2 \\ 0 & |x| > 2. \end{cases}$$

Finding its Fourier Transform, F(s), plot it.

(b) Solve the differential equation

$y''(t) + 4y(t) = 9t$ with initial condition $y(0) = 0$ and $y'(0) = 7$. (10)

7. (a) Show that the Dirac delta function can be expressed as the derivative of Heaviside's unit step function. (5)

(b) For the Dirac delta function $\delta(x)$, prove that

$$\delta(x^2 - a^2) = \frac{1}{2|a|}\left[\delta(x + |a|) + \delta(x - |a|)\right]. \quad (5)$$

P.T.O.

(c) If a continuous function f(t) is an even function, then show that its Fourier Transform F(ω) will also be an even function. (5)

8. (a) Find the Fourier Transform of the function

$$f(x) = e^{-\alpha|x|}, \qquad \alpha > 0$$

and hence show that

$$\int_{-\infty}^{\infty} \frac{e^{-ikx}}{(\alpha^2 + k^2)}\, dk = \frac{\pi}{\alpha}\, e^{-\alpha|x|}. \qquad (7)$$

(b) For a function

$$h(t) = \begin{cases} e^{-xt} g(t) & t > 0 \\ 0 & t < 0, \end{cases}$$

show that $F\{h(t)\} = L\{g(t)\}$. (3)

(c) For the equation $z^4 - 3z^2 + 1 = 0$, find the sum of its roots. (5)

[This question paper contains 6 printed pages.]

(16) Your Roll No. 2022

Sr. No. of Question Paper : 1379 A

Unique Paper Code : 32221402

Name of the Paper : Elements of Modern Physics

Name of the Course : **B.Sc. (Hons.) Physics**

Semester : IV – CBCS Part–II

Duration : 3.5 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Question no. **1** is compulsory.
3. Attempt **five** questions in all.
4. **All** questions carry equal marks.
5. Symbols have their usual meanings.

1. Attempt any **five** parts :

(a) If $f = x^n$, show that f is an eigen function of the operator $x\left(\frac{d}{dx}\right)$. Also find the eigenvalue.

P.T.O.

(b) David Beckham takes a free-kick of a football of mass 400 g. The curving ball moves with a velocity of 170 km/hr while reaching the goalpost. Find the deBroglie wavelength associated with the ball at that time. Will this wavelength have any physical significance for the goalkeeper facing the free-kick?

(c) Can the following two functions be physically acceptable solution of the Schrödinger wave equation

(i) (A/2) tan (x)

(ii) (3/2C) sin (x),

where A and C are non-zero constants.

(d) A 60 pm X-ray is incident on a calcite crystal. Find the wavelength of the X-rays scattered through an angle of 30°. What is the largest shift in wavelength that can be expected in this experiment?

(e) Find the combined kinetic energy of an electron and an antineutrino, when a free neutron decays into proton, electron and antineutrino. Given $m_n = 1.008984$ u, $m_p = 1.00759$ u, $m_e = 0.00055$ u, $1u = 1.673 \times 10^{-27}$ kg

(f) If ^{235}U loses 0.1% of its mass on undergoing fission, then how much energy is released when 1 Kg of ^{235}U undergoes fission?

(g) Why stimulated emission is necessary for lasing action? (5×3)

2. (a) The work function of potassium is 2.30 eV. UV light of wavelength 3000 Å and intensity $2Wm^{-2}$ is incident on the potassium surface.

(i) Determine the maximum kinetic energy of the photo electrons.

(ii) If 40% of incident photons produce photo electrons, how many electrons are emitted per second if the area of the potassium surface is 2 cm^2?

(b) The energy of a free electron including its rest mass energy is 10 MeV. Calculate the group velocity and the phase velocity of the wave packet associated with the motion of this electron.

(c) Deduce the Heisenberg's uncertainty principle for position and momentum from gamma ray microscope thought experiment. (5+5+5)

3. (a) Explain why it is plausible to define probability current density in quantum mechanics by the following expression

$$J = (-i\hbar/2m)\ (\psi^* \text{ grad } \psi - \psi \text{ grad } \psi^*)$$

The symbols have usual meaning.

(b) Name and explain an electron diffraction experiment. Give the physical significance of this experiment in relation to the wave particle duality. (10+5)

4. (a) Explain nuclear binding energy and packing fraction. Discuss graphically the variation of average binding energy per nucleon with mass number, A and hence explain nuclear stability and phenomena of fusion and fission.

(b) Calculate the binding energy per nucleon of $_{26}Fe^{56}$ in MeV using semi-empirical mass formula. Given $a_1 = 14.1$ MeV, $a_2 = 13.0$ MeV, $a_3 = 0.595$ MeV, $a_4 = 19.0$ MeV, $a_5 = 33.5$ MeV. (10+5)

5. A particle of mass m is confined in a field free region between impenetrable walls at x=0 and x=L.

(a) Obtain an expression for energy of the particle.

(b) Obtain and draw the first three normalized wave functions.

(c) Find the minimum energy of the particle with mass 9.1×10^{-31} kg for L=1 Å. (5+5+5)

6. (a) Given the half life of ^{210}Po is 138 days, find

(i) the decay constant of Po.

(ii) the activity of 1 g of Po.

(iii) how many decays per second occur when the sample is one week old.

(b) What are the main differences among alpha, beta and gamma decay?

(c) Name and explain which conservation laws seemed to be violated in beta decay. How did Pauli resolve these discrepancies? (5+5+5)

7. (a) What are the assumptions made in liquid drop model of atomic nucleus? How do asymmetry and pairing of the nucleons affect the nuclear stability?

(b) How are decay constant, half-life and average life time of a radioactive nuclide related with one another? Derive the equations connecting them. (10+5)

Some Physical Constants

Planck constant, $h = 6.626 \times 10^{-34} Js$

$h = 1.055 \times 10^{-34} Js$

Boltzmann constant, $K_B = 1.38 \times 10^{-23} JK^{-1}$

Mass of electron, $m_e = 9.1 \times 10^{-31}$ kg

Charge of electron, $e = 1.6 \times 10^{-19}$ C

Speed of light in vacuum, $c = 3 \times 10^8 ms^{-1}$

Stefan-Boltzmann constant, $\sigma = 5.67 \times 10^{-8} Wm^{-2}K^{-4}$

Rest mass energy of electron = 512 KeV

Velocity of electron in free space = $3 \times 10^8 ms^{-1}$

[This question paper contains 8 printed pages.]

17 **Your Roll No.** 2022

Sr. No. of Question Paper : 1397

A

Unique Paper Code : 32221403

Name of the Paper : Analog Systems and Applications

Name of the Course : **B.Sc. (Hons) Physics**

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

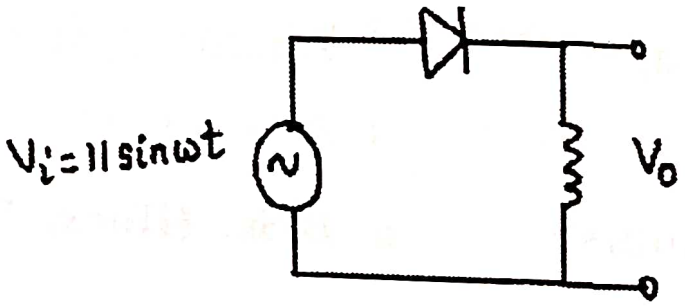
1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Question No. **1** is compulsory.
3. Attempt any **four** questions from the remaining **five** questions.
4. Non-programmable calculators are allowed.

1. Attempt any **five** of the following : (3×5=15)

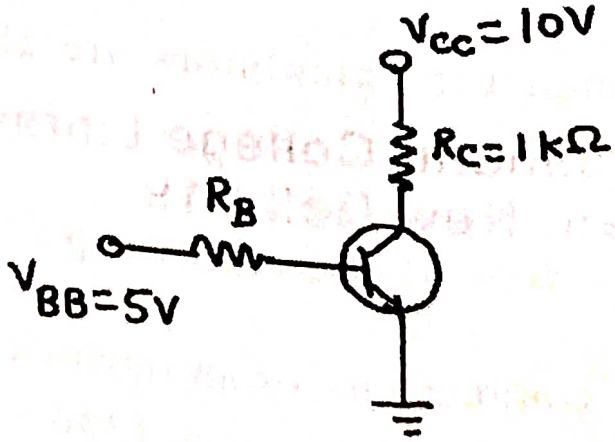
(a) Find the conductivity of an intrinsic silicon at 300K. Mobility of electrons $\mu_n = 1350\ cm^2/V\text{-}s$ and that of holes $\mu_p = 480\ cm^2/V\text{-}s$. Intrinsic concentrations of electrons and holes $n_i = 1.5 \times 10^{10} cm^{-3}$.

P.T.O.

(b) The output of the circuit given below is connected to the dc voltmeter. What is the reading on it? (Assume ideal diode).



(c) The transistor of the figure given below is specified to have β in the range 100 to 300. Find the value of R_B that results in saturation with an overdrive factor of at least 10. Assume $V_{CEsat} = 0.2V$ and $V_{BE} = 0.7V$.



(d) Design a differentiator to differentiate an input signal that varies in frequency from 10Hz to 1KHz, using op-amp.

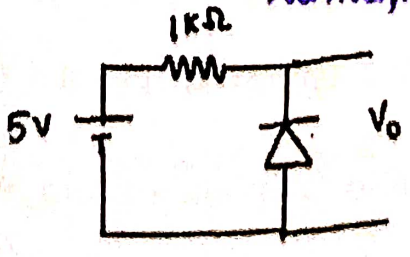
(e) Draw a circuit diagram of a 4-bit R-2R ladder type DAC and calculate its percentage resolution.

(f) Define slew rate and discuss why a high slew rate of an op-amp is desirable.

(g) Draw I-V characteristics of a Tunnel diode.

2. (a) A and B are two semiconductor materials. They have a band gap of 1.1 eV and 1.9 eV respectively. Which of these can be used for LED production? Support your answer by evaluating the wavelength of radiations emitted on recombination of electrons and holes in the two cases. Planck's constant $h = 6.626 \times 10^{-34}$ J s.

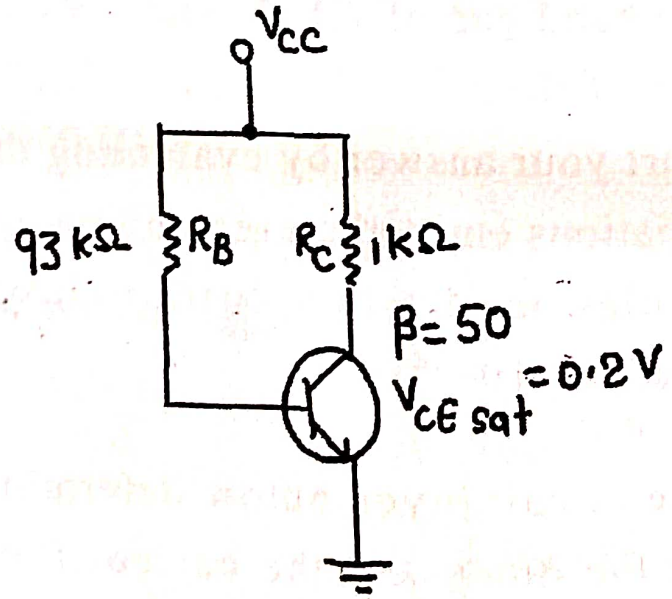
(b) For the circuit given below determine the voltage across the diode and the current flowing through it. Assume an ideal diode.



(c) Photodiodes and solar cells are both photovoltaic. What is the difference between the two?

(d) Explain Zener breakdown and discuss the main applications of Zener diode. (5,2,3,5)

3. (a) For the circuit given below, draw the load line and determine whether the transistor in the figure is in the active region or saturation region. What significant change will happen if the transistor is replaced by the one with double the value of β. (V_{CC}=10V, V_{BE}=0.7V and V_{CEsat}=0.2V).



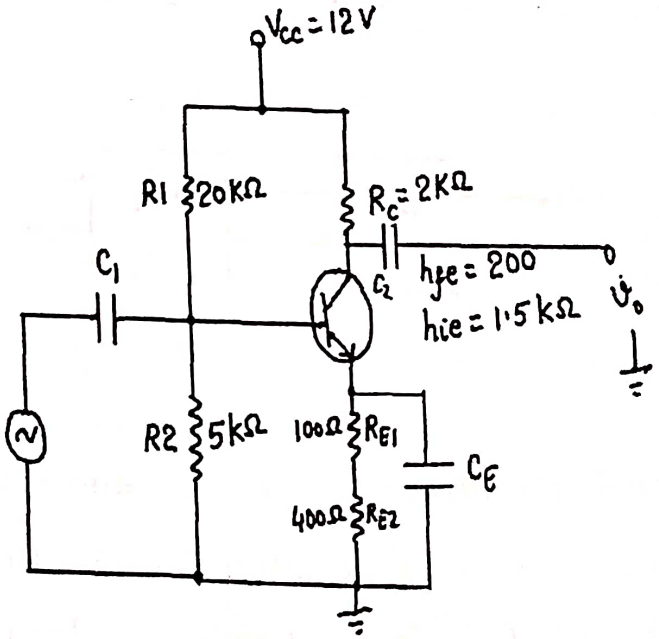
(b) In the circuit given below, evaluate :

(i) the operating point (V_{CE}, I_C)

(ii) mid frequency voltage gain

(iii) mid frequency voltage gain when bypass capacitor C_E is removed

(iv) mid frequency voltage gain when C_E is connected parallel to R_{E2}.



(5,10)

4. (a) Derive an expression for the frequency of oscillations and the condition for sustained oscillations for phase shift oscillator constructed using BJT.

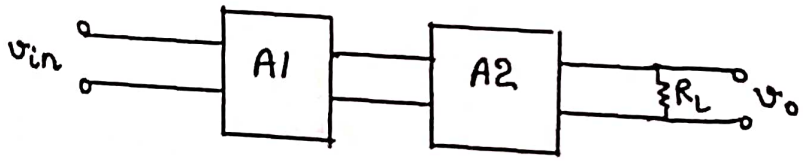
(b) A cascade connection of two voltage amplifiers A1 and A2 is shown in the figure given below. R_L=1kΩ. The open loop gain A_{V0}, input resistance

R_{IN} and output resistance R_0 for A1 and A2 are as follows:

A1: $A_{V0}=10$; $R_{IN}=10\ k\Omega$; $R_0=1k\Omega$

A2: $A_{V0}=5$; $R_{IN}=5\ k\Omega$; $R_0=200\Omega$.

What is the overall voltage gain?



(10,5)

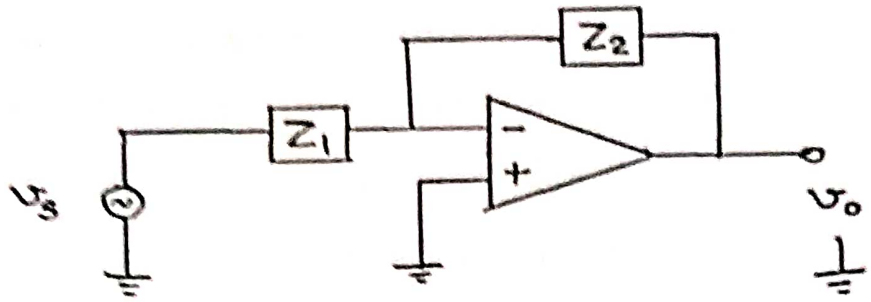
5. (a) The differential voltage gain and the common mode voltage gain of an operational amplifier are 100 dB and 2dB respectively. Calculate its CMRR. Why is it desirable to have high CMRR for an op-amp?

(b) Draw the frequency response of the gain for the circuit given below when:

(i) Z_1 and Z_2 are both resistors.

(ii) Z_1 is a resistor and Z_2 is a capacitor.

(iii) Z_1 is a capacitor and Z_2 is a resistor.



(c) Draw the circuit diagram of a basic integrator. Derive the expression for the output voltage. Discuss the problems associated with it. Also draw the circuit diagram of the practical integrator circuit that can integrate in the desired frequency range and rectify the problems associated with the basic circuit. (2,6,7)

6. (a) A silicon sample is doped with 10^{17} As atoms/cm^3. What is the equilibrium hole concentration p_0 at 300K? (The intrinsic electron and hole concentrations for silicon is $n_i = 1.5 \times 10^{10} cm^{-3}$).

(b) Calculate ripple factor and efficiency of a full wave rectifier. What is the PIV of a bridge rectifier?

(c) An op-amp is used as a zero-crossing detector. The maximum output available from the op-amp is

+12V & -12V and the slew rate of the op-amp is 12V/ps. What is the maximum frequency of the input signal that can be applied without causing distortion in the output?

(d) What will be the output of a comparator circuit if the inverting input terminal of the op-amp is connected to the ground and a sinusoidal voltage is applied to the non-inverting input terminal.

(3,7,3,2)